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Remarks on D_p & D_{p-2} with each carrying a flux

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Abstract

We explore various properties of interaction between D_p and $D_{p'}$ with each carrying a worldvolume flux and with the p'-branes placed along p' spatial worldvolume directions of the p-branes at a separation when $p - p' = 2$. Carefully analyzing the annulus amplitudes calculated via the boundary state approach, we find that many features of amplitudes remain similar to those studied when $p = p'$ such as the nature of force on the brane separation, the onset of various instabilities when the brane-separation is on the order of string scale and the occurrence of pair production of open strings when there is a relevant electric flux present. In addition, we have also found many new features of interaction which don't appear in the absence of fluxes or when $p = p'$ in the presence of fluxes, for examples, the nature of interaction can be repulsive and there is no onset of tachyonic instability under certain conditions. Even in the absence of a magnetic flux, we can have an exponential enhancement of the rate of pair production of open strings in certain cases, which may be significant enough to give observational consequences.

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It is known that the static interaction between a D_p and a $\mathrm{D}_{p'}$ brane at a separation with $p - p' = 2$ is always attractive[3] when neither carries a world-volume flux, for example, see [1]. This nature of interaction is expected to be modified or changed when there is an electric or a magnetic flux present on either of the branes[4]. In this note, we will explore this modification or change and the associated properties such as the onset of various instabilities and the open string pair production, expecting some unique and interesting physical implications to arise. We find that the pair production rate is significant even for the mere presence of a weak electric flux under certain condition. Unlike the rate enhancement discussed in [2], an additional magnetic flux not sharing any common index with the electric flux is not necessarily needed. The novel feature here is that the D_{p-2} brane itself takes the role of the magnetic flux to enhance the rate. The magnetic flux can further either enhance or reduce the rate, depending on the actual case considered. One interesting observation is that the largest rate occurs either for $p = 3$ as also observed in [2], for which $p = p'$ was considered, or for $p' = 3$. We don't know if this has an implication of why our world has three large spatial dimensions and even for the existence of extra dimensions. When both fluxes are magnetic, the would-be attractive interaction can vanish in certain cases under conditions specified later, signalling the preservation of certain number of supersymmetries (susy) of the underlying systems. When both these fluxes point to different NN directions, these is an interesting case for which the force can even be repulsive for certain range of the fluxes. For this case and the above susy cases, the corresponding amplitude doesn't give rise to a tachyonic instability, therefore no tachyon condensation to occur.

Without further ado, we have the following three cases to consider[5]: 1) D_p and $\mathrm{D}_{p'}$ carry their respective electric fluxes F_{0a} and F'_{0b}; 2) D_p and $\mathrm{D}_{p'}$ carry their respective magnetic fluxes F_{ab} and F'_{cd}; 3) one carries an electric flux and the other carries a magnetic flux. Or we can classify the interaction amplitudes according to their structures determined by the relative orientations of the two fluxes $F_{\alpha\beta}$ and $F'_{\gamma\delta}$ in the following three classes: I) the indices $\alpha, \beta, \gamma, \delta \in$ NN with the pair $(\alpha,\ \beta)$ and the pair $(\gamma,\ \delta)$ sharing at least one common index (either temporal or spatial) or the index α or $\beta \in$ ND but not

[3]In other words, we consider in this paper the case of $p - p' = 2$ with $NN = p' + 1, ND = 2$ and $DD = 9 - p$. The D_p and $\mathrm{D}_{p'}$ are placed at a separation along the $DD = 9 - p$ directions with $p \leq 8$.

[4]As mentioned in [3, 2] and the references therein, D_k branes become a 1/2−Bogomol'nyi-Prasad-Sommerfield (BPS) non-threshold (F, D_k) bound state when carrying an electric flux while a 1/2-BPS non-threshold (D_{k-2}, D_k) bound state when carrying a magnetic flux.

[5]In this note, the Greek indices $\alpha = (0, a), \beta = (0, b), \cdots$ label the world-volume directions $0, 1, \cdots, p$ along which the D_p brane extends with $a, b, \cdots$ denoting the brane spatial diretions, while the later Latin indices $i, j, \cdots$ label the directions transverse to the brane, i.e., $p + 1, \cdots, 9$.

both; II) $\alpha, \beta, \gamma, \delta \in$ NN but $\alpha, \beta \neq \gamma, \delta$; III) $\alpha, \beta \in$ ND and $\gamma, \delta \in$ NN. Note that we have $p \geq 3$ for Class I or III and $p \geq 5$ for Class II. We will present the amplitudes according to this classification for simplicity and for a unified description in each class. Note that we have non-vanishing contributions only from the Neveu-Schwarz−Neveu-Schwarz (NS-NS) sector for amplitudes in Class I or Class II but this is not true for amplitudes in Class III.

The tree-level cylinder diagram interaction can be calculated via the boundary state approach. As usual, we have two sectors, namely NS-NS and Ramond−Ramond (R-R) sectors, respectively. We here summarize the main results needed, following [5]. Let F be the external flux on the world-volume and denote $\hat{F} = 2\pi\alpha' F$. In the NS-NS sector, the relevant boundary state is the Gliozzi-Scherk-Olive (GSO) projected one $|B\rangle_{NS} = \frac{1}{2}\left[|B,+\rangle_{NS} - |B,-\rangle_{NS}\right]$ while in the R-R sector, the GSO projected one is $|B\rangle_R = \frac{1}{2}\left[|B,+\rangle_R + |B,-\rangle_R\right]$. Here the two boundary states $|B,\eta\rangle$ with $\eta = \pm$ correspond to two possible implementations for the boundary conditions in each sector. The boundary state $|B,\eta\rangle$ is the product of a matter part and a ghost part as $|B,\eta\rangle = \frac{c_p}{2}|B_{\rm mat.},\eta\rangle|B_g,\eta\rangle$ with $|B_{\rm mat.},\eta\rangle = |B_X\rangle|B_\psi,\eta\rangle$, $|B_g,\eta\rangle = |B_{gh}\rangle|B_{\rm sgh},\eta\rangle$ and the normalization constant $c_p = \sqrt{\pi}\left(2\pi\sqrt{\alpha'}\right)^{3-p}$. The explicit forms of the various components of $|B\rangle$ are given as $|B_X\rangle = \exp[-\sum_{n=1}^\infty \frac{1}{n}\alpha_{-n}\cdot S\cdot\tilde{\alpha}_{-n}]|B_X\rangle^{(0)}$, and for the NS-NS sector $|B_\psi,\eta\rangle_{\rm NS} = -{\rm i}\,\exp[{\rm i}\,\eta\sum_{m=1/2}^\infty \psi_{-m}\cdot S\cdot\tilde{\psi}_{-m}]|0\rangle$ and for the R-R sector $|B_\psi,\eta\rangle_{\rm R} = -\exp[{\rm i}\,\eta\sum_{m=1}^\infty \psi_{-m}\cdot S\cdot\tilde{\psi}_{-m}]|B,\eta\rangle_{\rm R}^{(0)}$. The matrix S and the zero-mode contributions $|B_X\rangle^{(0)}$ and $|B,\eta\rangle_{\rm R}^{(0)}$ encode all information about the external flux and the overlap equations that the string coordinates have to satisfy, which in turn depend on the boundary conditions of the open strings ending on the D-brane. They each can be given explicitly as $S = ([(\eta-\hat{F})(\eta+\hat{F})^{-1}]_{\alpha\beta}, -\delta_{ij})$, $|B_X\rangle^{(0)} = \sqrt{-\det(\eta+\hat{F})}\,\delta^{9-p}(q^i - y^i)\prod_{\mu=0}^9|k^\mu = 0\rangle$, and $|B_\psi,\eta\rangle_{\rm R}^{(0)} = (C\Gamma^0\Gamma^1\cdots\Gamma^p\frac{1+{\rm i}\,\eta\Gamma_{11}}{1+{\rm i}\,\eta}U)_{AB}|A\rangle|\tilde{B}\rangle$. In the above, we have denoted by y^i the positions of the D-brane along the transverse directions, by C the charge conjugation matrix and by U the following matrix $U = ;\exp(-\frac{1}{2}\hat{F}_{\alpha\beta}\Gamma^\alpha\Gamma^\beta);\,/\sqrt{-\det(\eta+\hat{F})}$ where the symbol ; ; means that one has to expand the exponential and then to anti-symmetrize the indices of the Γ-matrices. $|A\rangle|\tilde{B}\rangle$ stands for the spinor vacuum of the R-R sector. Note that the η in the above means either sign $\pm$ or the flat signature matrix $(-1,+1,\cdots,+1)$ on the world-volume and should not be confused from the content. Note also that the boundary state must be written in the $(-1,-1)$ super-ghost picture in the NS-NS sector, and in the asymmetric $(-1/2,-3/2)$ picture in the R-R sector in order to saturate the super-ghost number anomaly of the disk.

The interaction under consideration can be calculated as the vacuum amplitude of the closed string tree-level cylinder diagram via the boundary states as just described. The

total amplitude can have contributions from both NS-NS sector and R-R sector and is given by $\Gamma = \Gamma_{\rm NS} + \Gamma_{\rm R}$ with $\Gamma_{\rm NS/R} = {}_{\rm NS/R}\langle B_{p-2}|D|B_p\rangle\rangle_{\rm NS/R}$. Here D is the closed string propagator

$$D = \frac{\alpha'}{4\pi}\int_{|z|\leq 1}\frac{d^2 z}{|z|^2} z^{L_0}\bar{z}^{\tilde{L}_0}, \tag{1}$$

with L_0 and $\tilde{L}_0$ the respective left and right moving total zero-mode Virasoro generators of matter fields, ghosts and superghosts. For example, $L_0 = L_0^X + L_0^\psi + L_0^{gh} + L_0^{sgh}$ and their explicit expressions can be found, e.g., from [3]. The above amounts to calculating first the following amplitude in the respective sector

$$\Gamma(\eta',\eta) = \langle B_{p-2},\eta'|D|B_p,\eta\rangle = \frac{n_{p-2}n_p c_{p-2}c_p}{4}\frac{\alpha'}{4\pi}\int_{|z|\leq 1}\frac{d^2 z}{|z|^2} A^X\, A^{bc}\, A^\psi(\eta',\eta)\, A^{\beta\gamma}(\eta',\eta) \tag{2}$$

where $\eta'\eta = \pm$ and we have also replaced the c_k $(k = p-2\,{\rm or}\, p)$ in the boundary state given earlier by $n_k c_k$ with n_k an integer to count the multiplicity of the D_k branes in the bound state. In the above, the various matrix elements are

$$\begin{aligned} A^X &= \langle B^X_{p-2}||z|^{2L_0^X}|B^X_p\rangle, \quad A^\psi(\eta',\eta) = \langle B^\psi_{p-2},\eta'||z|^{2L_0^\psi}|B^\psi_p,\eta\rangle,\\ A^{bc} &= \langle B^{gh}_{p-2}||z|^{2L_0^{bc}}|B^{gh}_p\rangle, \quad A^{\beta\gamma}(\eta',\eta) = \langle B^{sgh}_{p-2},\eta'||z|^{2L_0^{\beta\gamma}}|B^{sgh}_p,\eta\rangle. \end{aligned} \tag{3}$$

where we have used the boundary state constraint $\tilde{L}_0|B\rangle = L_0|B\rangle$ to simplify the calculations. Note that the A^{bc} and $A^{\beta\gamma}$ are independent of fluxes and are always given as $A^{bc} = |z|^{-2}\prod_{n=1}^\infty(1-|z|^{2n})^2$ in both NS-NS and R-R sectors, and in the NS-NS sector, $A^{\beta\gamma}_{\rm NS}(\eta',\eta) = |z|\prod_{n=1}^\infty(1+\eta'\eta|z|^{2n-1})^{-2}$ while in the R-R sector, $A^{\beta\gamma}_{\rm R}(\eta',\eta) = |z|^{3/4}(1+\eta'\eta)^{-1}\prod_{n=1}^\infty(1-\eta'\eta|z|^{2n})^{-2}$.

However in what follows, we will adopt the prescription given in [6, 1] not to separate the contributions from matter fields ψ^μ and superghosts in the R-R sector to avoid the complication due to the respective zero modes if this sector has non-zero contribution. To calculate A^X and $A^\psi(\eta',\eta)$, we will follow the trick as described in [4, 2] by making a respective unitary transformation of the oscillators in $|B^{X/\psi}_{p-2},\eta'\rangle$ such that the S_{p-2}-matrix there completely disappears while $|B^{X/\psi}_p,\eta\rangle$ ends up with a new $S = S_p S^T_{p-2}$ with S_p the original S-matrix in this boundary state and T denoting the transpose. This new S-matrix shares the same property as the original S_k satisfying $(S_k^T)_\mu{}^\rho(S_k)_\rho{}^\nu = \delta_\mu{}^\nu$ with $k = p-2\,{\rm or}\,p$ but its determinant is always unity and therefore can always be diagonalized to gives its eigenvalues. With this trick, the evaluation of $A^{X/\psi}$ is no more complicated than the case without the presence of fluxes.

If we take $(\hat{F}')_{\gamma\delta} = -(\hat{F}')_{\delta\gamma} = -f_1$ with $\gamma < \delta$ and $(\hat{F})_{\alpha\beta} = -(\hat{F})_{\beta\alpha} = -f_2$ with

$\alpha < \beta$, the Class I matrix elements for matter fields are

$$A^X = C_F V_{p-1} e^{-\frac{Y^2}{2\pi\alpha' t}} \left(2\pi^2\alpha'\, t\right)^{-\frac{9-p}{2}} \prod_{n=1}^{\infty} \frac{1}{(1-\lambda|z|^{2n})(1-\lambda^{-1}|z|^{2n})(1+|z|^{2n})^2(1-|z|^{2n})^6} \tag{4}$$

for both NS-NS and R-R sectors,

$$A^{\psi}_{NS}(\eta',\eta) = \prod_{n=1}^{\infty}(1+\eta'\eta\lambda|z|^{2n-1})(1+\eta'\eta\lambda^{-1}\,|z|^{2n-1})(1-\eta'\eta|z|^{2n-1})^2(1+\eta'\eta|z|^{2n-1})^6 \tag{5}$$

for the NS-NS sector, and $A^{\psi}_{R}(\eta',\eta) = 0$ for the R-R sector. Note that $\Gamma_{\rm R} = 0$, so the total amplitude is just

$$\Gamma_{\rm I} = \Gamma_{\rm NS} \;=\; \frac{2n_{p-2}\, n_p\, V_{p-1}\, C_F\, \sin\pi\nu}{(8\pi^2\alpha')^{\frac{p-1}{2}}} \int_0^{\infty} dt\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{9-p}{2}}\, \frac{\theta_1^2(\frac{2\nu-1}{4}|it)\,\theta_1^2(\frac{2\nu+1}{4}|it)}{\eta^6(it)\,\theta_1(\nu|it)\,\theta_1(\frac{1}{2}|it)}, \tag{6}$$

where we have taken $|z| = e^{-\pi t}$ and $\lambda = e^{2i\pi\nu}$. In obtaining the above compact expression, we have first expressed the integrand in terms of various θ-functions and the Dedekind η-function, then made use of the fundamental Jacobian identity $2\,\theta_1^2(\frac{2\nu-1}{4}|it)\,\theta_1^2(\frac{2\nu+1}{4}|it) = \theta_3(\nu|it)\,\theta_3(\frac{1}{2}|it)\,\theta_3^2(0|it) - \theta_4(\nu|it)\,\theta_4(\frac{1}{2}|it)\,\theta_4^2(0|it)$, which is a special form of (iv) given on page 468 in [7]. The constant C_F and the sum[6] of $\lambda + \lambda^{-1}$ can be summarized for cases considered in this class in Table 1.

The Class II matrix elements for matter fields are

$$A^X = C_F V_{p-1} e^{-\frac{Y^2}{2\pi\alpha' t}} \left(2\pi^2\alpha'\, t\right)^{-\frac{9-p}{2}} \prod_{n=1}^{\infty} \frac{1}{(1+|z|^{2n})^2(1-|z|^{2n})^4} \prod_{j=1}^{2} \frac{1}{(1-\lambda_j|z|^{2n})(1-\lambda_j^{-1}|z|^{2n})} \tag{7}$$

for both NS-NS and R-R sectors,

$$A^{\psi}_{NS}(\eta',\eta) = \prod_{n=1}^{\infty}(1-\eta'\eta|z|^{2n-1})^2(1+\eta'\eta|z|^{2n-1})^4 \prod_{j=1}^{2}(1+\eta'\eta\lambda_j|z|^{2n-1})(1+\eta'\eta\lambda_j^{-1}\,|z|^{2n-1}) \tag{8}$$

for the NS-NS sector, and again $A^{\psi}_{R}(\eta',\eta) = 0$ for the R-R sector. Once again $\Gamma_{\rm R} = 0$ and the total amplitude in this Class is

$$\begin{aligned}\Gamma_{\rm II} = \Gamma_{\rm NS} \;&=\; \frac{4n_{p-2}n_pV_{p-1}\tan\pi\nu_1\tan\pi\nu_2}{(8\pi^2\alpha')^{\frac{p-1}{2}}} \int_0^{\infty} dt e^{-\frac{y^2}{2\pi\alpha' t}} t^{-\frac{9-p}{2}} \\ &\quad\times \frac{\theta_1(\frac{\nu_1-\nu_2-1/2}{2}|it)\theta_1(\frac{\nu_1-\nu_2+1/2}{2}|it)\theta_1(\frac{\nu_1+\nu_2-1/2}{2}|it)\theta_1(\frac{\nu_1+\nu_2+1/2}{2}|it)}{\eta^3(it)\theta_1(\nu_1|it)\theta_1(\nu_2|it)\theta_1(\frac{1}{2}|it)},\end{aligned} \tag{9}$$

[6] We need only this sum to determine ν via $\cos\pi\nu$ in terms of flux/fluxes given in the table. When the flux/fluxes are electric or electrically dominant, the ν is imaginary and $\cos\pi\nu$ in the table represents actually $\cosh\pi\nu_0$ since $\cos\pi\nu = \cosh\pi\nu_0$ when $\nu = i\nu_0$. This remains also true in Table 2 and 3.

Table 1: The cases in Class I

(α,β)	(γ,δ)	Index Relation	C_F	$\frac{\lambda+\lambda^{-1}}{2}$	$\cos\pi\nu$
$(0,a)$	$(0,c)$	$a=c\in$ NN	$\sqrt{(1-f_1^2)(1-f_2^2)}$	$\frac{(1+f_1^2)(1+f_2^2)-4f_1f_2}{(1-f_1^2)(1-f_2^2)}$	$\frac{1-f_1f_2}{C_F}$
$(0,a)$	$(0,c)$	$a,c\in$ NN, $a\neq c$	$\sqrt{(1-f_1^2)(1-f_2^2)}$	$\frac{1+f_1^2+f_2^2-f_1^2f_2^2}{(1-f_1^2)(1-f_2^2)}$	$\frac{1}{C_F}$
$(0,a)$	$(0,c)$	$a\in$ DN, $c\in$ NN	$\sqrt{(1-f_1^2)(1-f_2^2)}$	$\frac{1+f_1^2-f_2^2+f_1^2f_2^2}{(1-f_1^2)(1-f_2^2)}$	$\frac{[1-f_2^2(1-f_1^2)]^{\frac{1}{2}}}{C_F}$
$(0,a)$	(c,d)	$a,c,d\in$ NN, $a=c$ or d	$\sqrt{(1+f_1^2)(1-f_2^2)}$	$\frac{1-f_1^2+f_2^2+f_1^2f_2^2}{(1+f_1^2)(1-f_2^2)}$	$\frac{1}{C_F}$
$(0,a)$	(c,d)	$a\in$ DN, $c,d\in$ NN	$\sqrt{(1+f_1^2)(1-f_2^2)}$	$\frac{1-f_1^2}{1+f_1^2}$	$\frac{1}{\sqrt{1+f_1^2}}$
(a,b)	$(0,c)$	$a,b,c\in$ NN, $c=a$ or b	$\sqrt{(1-f_1^2)(1+f_2^2)}$	$\frac{1+f_1^2-f_2^2+f_1^2f_2^2}{(1-f_1^2)(1+f_2^2)}$	$\frac{1}{C_F}$
(a,b)	$(0,c)$	$c=a\in$ NN, $b\in$ DN	$\sqrt{(1-f_1^2)(1+f_2^2)}$	$\frac{1+f_1^2+f_2^2-f_1^2f_2^2}{(1-f_1^2)(1+f_2^2)}$	$\frac{[1+f_2^2(1-f_1^2)]^{\frac{1}{2}}}{C_F}$
(a,b)	$(0,c)$	$c,a\in$ NN, $b\in$ DN, $c\neq a$	$\sqrt{(1-f_1^2)(1+f_2^2)}$	$\frac{1+f_1^2}{1-f_1^2}$	$\frac{1}{\sqrt{1-f_1^2}}$
(a,b)	(c,d)	$a,b,c,d\in$ NN, $a=c,b=d$	$\sqrt{(1+f_1^2)(1+f_2^2)}$	$\frac{(1-f_1^2)(1-f_2^2)+4f_1f_2}{(1+f_1^2)(1+f_2^2)}$	$\frac{\lvert 1+f_1f_2\rvert}{C_F}$
(a,b)	(c,d)	$a,b,c,d\in$ NN, $a=c,b\neq d$, or $a=d$	$\sqrt{1+f_1^2)(1+f_2^2)}$	$\frac{1-f_1^2-f_2^2-f_1^2f_2^2}{(1+f_1^2)(1+f_2^2)}$	$\frac{1}{C_F}$
(a,b)	(c,d)	$a,c,d\in$ NN, $b\in$ DN, $a=c$ or d	$\sqrt{1+f_1^2)(1+f_2^2)}$	$\frac{1-f_1^2+f_2^2+f_1^2f_2^2}{(1+f_1^2)(1+f_2^2)}$	$\frac{[1+f_2^2(1+f_1^2)]^{\frac{1}{2}}}{CF}$
(a,b)	(c,d)	$a,c,d\in$ NN, $b\in$ DN, $a\neq c,d$	$\sqrt{1+f_1^2)(1+f_2^2)}$	$\frac{1-f_1^2}{1+f_1^2}$	$\frac{1}{\sqrt{1+f_1^2}}$

where we have taken $|z|=e^{-\pi t}$ and $\lambda_j=e^{2i\pi\nu_j}$ with $j=1,2$. Similarly, we first express the integrand via various θ-functions and the Dedekind η-function, then use another fundamental Jacobian identity $2\,\theta_1(\frac{\nu_1-\nu_2-1/2}{2}|it)\theta_1(\frac{\nu_1-\nu_2+1/2}{2}|it)\,\theta_1(\frac{\nu_1+\nu_2-1/2}{2}|it)\,\theta_1(\frac{\nu_1+\nu_2+1/2}{2}|it)=\theta_3(\nu_1|it)\,\theta_3(\nu_2|it)\,\theta_3(1/2|it)\,\theta_3(0|it)-\theta_4(\nu_1|it)\,\theta_4(\nu_2|it)\,\theta_4(1/2|it)\,\theta_4(0|it)$, which is also a special form of (iv) given on page 468 in [7]. The constant C_F and the sum of $\lambda_j+\lambda_j^{-1}$ for the cases considered in this class are listed in Table 2.

The Class III matrix elements for matter fields are

$$A^X=C_FV_{p-1}e^{-\frac{Y^2}{2\pi\alpha' t}}\left(2\pi^2\alpha'\,t\right)^{-\frac{9-p}{2}}\prod_{n=1}^{\infty}\frac{1}{(1-|z|^{2n})^6}\prod_{j=1}^{2}\frac{1}{(1-\lambda_j|z|^{2n})(1-\lambda_j^{-1}|z|^{2n})}\qquad(10)$$

Table 2: The cases in Class II

(α,β)	(γ,δ)	Index relation	C_F	$\frac{\lambda_1+\lambda_1^{-1}}{2}$	$\frac{\lambda_2+\lambda_2^{-1}}{2}$	$\cos\pi\nu_1$	$\cos\pi\nu_2$
(a,b)	$(0,c)$	$a,b,c\in$ NN, $c\neq a,b$	$\sqrt{(1-f_1^2)(1+f_2^2)}$	$\frac{1+f_1^2}{1-f_1^2}$	$\frac{1-f_2^2}{1+f_2^2}$	$\frac{1}{(1-f_1^2)^{\frac{1}{2}}}$	$\frac{1}{(1+f_2^2)^{\frac{1}{2}}}$
$(a,0)$	(c,d)	$a,c,d\in$ NN, $a\neq c,d$	$\sqrt{(1+f_1^2)(1-f_2^2)}$	$\frac{1-f_1^2}{1+f_1^2}$	$\frac{1+f_2^2}{1-f_2^2}$	$\frac{1}{(1+f_1^2)^{\frac{1}{2}}}$	$\frac{1}{(1-f_2^2)^{\frac{1}{2}}}$
(a,b)	(c,d)	$a,b,c,d\in$ NN, $a,b\neq c,d$	$\sqrt{(1+f_1^2)(1+f_2^2)}$	$\frac{1-f_1^2}{1+f_1^2}$	$\frac{1-f_2^2}{1+f_2^2}$	$\frac{1}{(1+f_1^2)^{\frac{1}{2}}}$	$\frac{1}{(1+f_2^2)^{\frac{1}{2}}}$

for both NS-NS and R-R sectors,

$$A^{\psi}_{NS}(\eta',\eta)=\prod_{n=1}^{\infty}(1+\eta'\eta|z|^{2n-1})^6\prod_{j=1}^{2}(1+\eta'\eta\lambda_j|z|^{2n-1})(1+\eta'\eta\lambda_j^{-1}|z|^{2n-1}) \tag{11}$$

for the NS-NS sector, and

$$A^{\psi}_{R}(\eta',\eta)A^{\beta\gamma}_{R}(\eta',\eta)=-(\sqrt{2})^8|z|^2D_F\delta_{\eta'\eta,+}\prod_{n=1}^{\infty}(1+|z|^{2n})^4\prod_{j=1}^{2}(1+\lambda_j|z|^{2n})(1+\lambda_j^{-1}|z|^{2n}) \tag{12}$$

for the R-R sector. Note that now the R-R sector has non-zero contribution and we don't separate the contributions from the matter field ψ^μ and the superghosts as mentioned earlier. The total amplitude is now

$$\Gamma_{\rm III}=\Gamma_{\rm NS}+\Gamma_{\rm R} \;=\; \frac{2n_{p-2}\,n_p\,V_{p-1}\tan\pi\nu_1}{(8\pi^2\alpha')^{\frac{p-1}{2}}}\int_0^\infty dte^{-\frac{y^2}{2\pi\alpha' t}}t^{-\frac{9-p}{2}}\frac{\theta_1^2(\frac{\nu_1-\nu_2}{2}|it)\theta_1^2(\frac{\nu_1+\nu_2}{2}|it)}{\eta^6(it)\theta_1(\nu_1|it)\theta_1(\nu_2|it)} \tag{13}$$

where again $|z|=e^{-\pi t}$ and $\lambda_j=e^{2i\pi\nu_j}$ with $j=1,2$. Once again in obtaining the above compact expression, we have expressed the integrand in terms of various θ-functions and the Dedekind η-function, then used yet another fundamental Jacobian identity $2\,\theta_1^2(\frac{\nu_1-\nu_2}{2}|it)$ $\theta_1^2(\frac{\nu_1+\nu_2}{2}|it)=\theta_3(\nu_1|it)\,\theta_3(\nu_2|it)\,\theta_3^2(0|it)-\theta_4(\nu_1|it)\,\theta_4(\nu_2|it)\,\theta_4^2(0|it)-\theta_2(\nu_1|it)\,\theta_2(\nu_2|it)\,\theta_2^2(0|it)$ where the first two terms come from the NS-NS sector and the last term comes from the R-R sector. This identity is also a special form of (iv) given on page 468 in [7]. The constants C_F and the sum of $\lambda_j+\lambda_j^{-1}$ for cases considered in this class are listed in Table 3 with $D_F=-f_2/C_F=\cos\pi\nu_1\cos\pi\nu_2$.

We now come to discuss the nature and range of ν's for cases in each class which will depend crucially on the nature of fluxes (electric or magnetic) involved as discussed in [4, 2]. Note that for an electric flux f, $0<|f|<1$ with the critical field $|f|=1$ while for

Table 3: The cases in Class III

(α,β)	(γ,δ)	Index Relation	C_F	$\frac{\lambda_1+\lambda_1^{-1}}{2}$	$\frac{\lambda_2+\lambda_2^{-1}}{2}$	$\cos\pi\nu_1$	$\cos\pi\nu_2$
(a,b)	$(0,c)$	$c\in$ NN, $a,b\in$ DN	$\sqrt{(1-f_1^2)(1+f_2^2)}$	$\frac{1+f_1^2}{1-f_1^2}$	$-\frac{1-f_2^2}{1+f_2^2}$	$\frac{1}{(1-f_1^2)^{\frac12}}$	$-\frac{f_2}{(1+f_2^2)^{\frac12}}$
(a,b)	(c,d)	$c,d\in$ NN, $a,b\in$ DN	$\sqrt{(1+f_1^2)(1+f_2^2)}$	$\frac{1-f_1^2}{1+f_1^2}$	$-\frac{1-f_2^2}{1+f_2^2}$	$\frac{1}{(1+f_1^2)^{\frac12}}$	$-\frac{f_2}{(1+f_2^2)^{\frac12}}$

a magnetic flux f, $0<|f|<\infty$. In Class I, when both fluxes are electric, ν is imaginary, i.e., $\nu=i\nu_0$ and $\cos\pi\nu=\cosh\pi\nu_0$ with $0<\nu_0<\infty$ (see footnote 6 for detail). When the two are both magnetic, $\nu=\nu_0$ is real with $0<\nu_0<1/2$. When one flux is electric and the other magnetic, we have the following cases: 1) $(\hat F)_{0a}$ and $(\hat F')_{cd}$ with $a\in$ DN, $c,d\in$ NN, then $\nu=\nu_0$ is real with $0<\nu_0<1/2$; 2) $(\hat F)_{ab}$ and $(\hat F')_{0c}$ with $c,a\in$ NN, $b\in$ ND, then $\nu=i\nu_0$ (also $\cos\pi\nu=\cosh\pi\nu_0$) is imaginary with $0<\nu_0<\infty$; 3) $(\hat F)_{0a}$ and $(\hat F')_{cd}$ with $a,c,d\in$ NN, $a=c\,\mathrm{or}\,d$ (or $(\hat F)_{ab}$ and $(\hat F')_{0c}$ with $a,b,c\in$ NN, $c=a\,\mathrm{or}\,b$), $\cos\pi\nu=1/\sqrt{(1+f_1^2)(1-f_2^2)}$ (or $=1/\sqrt{(1-f_1^2)(1+f_2^2)}$) where ν can be non-vanishing real, imaginary, or zero, depending on $\sqrt{(1-f_1^2)(1+f_2^2)}$(or $\sqrt{(1+f_1^2)(1-f_2^2)}$) $>1,<1$ or $=1$, respectively, for non-vanishing fluxes[7]. For Class II, the nature of ν_j $(j=1,2)$ is directly related to the corresponding flux and $\nu_j=i\nu_{j0}$ is imaginary with $0<\nu_{j0}<\infty$ if the corresponding flux is electric and $\nu_j=\nu_{j0}$ is real with $0<\nu_{j0}<1/2$ if the flux is magnetic. For Class III, while ν_1 remains the same nature as the ν_j in Class II, $\nu_2=\nu_{20}$ is however always real with now $0<\nu_{20}<1$ for which $0<\nu_{20}<1/2$ corresponds to $f_2<0$, $\nu_{20}=1/2$ to $f_2=0$ and $1/2<\nu_{20}<1$ to $f_2>0$.

Let us first consider the large-separation limit of the above amplitudes for $2\le p\le 6$. This amounts to taking the large t-limit for the θ_1-function and the Dedekind η-function in each integrand, i.e., $\theta_1(\nu|it)\to 2\,e^{-\frac{\pi t}{4}}\sin\pi\nu$, $\eta(it)\to e^{-\frac{\pi}{12}t}$ (noting now $|z|=e^{-\pi t}\to 0$). We then have $\Gamma_{\rm I}\to C_{\rm I}/Y^{7-p}$ where a simple integration has been performed and $C_{\rm I}=n_p\,n_{p-2}\,c_p\,c_{p-2}\,V_{p-1}C_F(\lambda+\lambda^{-1}+2)/[4(7-p)\Omega_{8-p}]$ with $(7-p)\Omega_{8-p}=4\pi\pi^{(7-p)/2}/\Gamma((7-p)/2)$ and Ω_q the volume of unit q-sphere. Similarly, we have $\Gamma_{\rm II}\to C_{\rm II}/Y^{7-p}$ with $C_{\rm II}=n_pn_{p-2}\,c_pc_{p-2}\,V_{p-1}C_F(\lambda_1+\lambda_1^{-1}+\lambda_2+\lambda_2^{-1})/[4(7-p)\Omega_{8-p}]$, and $\Gamma_{\rm III}\to C_{\rm III}/Y^{7-p}$ with $C_{\rm III}=n_pn_{p-2}\,c_pc_{p-2}\,V_{p-1}C_F\,(\lambda_1+\lambda_1^{-1}+\lambda_2+\lambda_2^{-1}+4-8D_F)/[4(7-p)\Omega_{8-p}]$. Note that the parameters λ, or $\lambda_j(j=1,2)$, C_F(and D_F) in each relevant class are given in the respective table and $c_k=\sqrt{\pi}(2\pi\sqrt{\alpha'})^{3-k}$ with $k=p,p-2$.

Before proceeding, we pause to discuss which amplitude can vanish when either flux is

[7]Here when ν is real, we call it magnetically dominant otherwise electrically dominant.

non-critical and non-vanishing. Note that the vanishing large-separation amplitude also implies the vanishing of the corresponding amplitude at a general separation. This can possibly occur only for one case in each class and only when both fluxes are magnetic. Concretely, the amplitude vanishes in Class I for the case of $a, b, c, d \in \mathrm{NN}, a, b = c, d$ (see Table 1) when $f_1 f_2 = -1$ (corresponding to $\nu = 1/2$), in Class II for the case of $a, b, c, d \in \mathrm{NN}, a, b \neq c, d$ (see Table 2) when $f_1 f_2 = \pm 1$ (corresponding to $\nu_1 + \nu_2 = 1/2$), and in Class III for the case of $a, b \in \mathrm{DN}, c, d \in \mathrm{NN}$ (see Table 3) when $f_1 f_2 = \pm 1$ and $f_2 < 0$ (corresponding to $\nu_1 = \nu_2$). The vanishing of amplitude also implies the preservation of certain number of supersymmetries for the underlying system which can be similarly analyzed following the steps given in the appendix of [2]. Here we simply state the results: the case in Class I or III preserves 1/4 of the spacetime supersymmetries while in Class II it preserves only 1/8 of the supersymmetries.

Apart from the above three special cases and one case associated with the special case in Class II, all the other large-separation amplitudes are positive, therefore giving attractive interactions. This particular case corresponds to the one given in Table 2 in Class II when both fluxes are magnetic with their respective spatial indices $a, b, c, d \in \mathrm{NN}$; $a, b \neq c, d$ and $f_1^2 f_2^2 > 1$ (corresponding to $1/2 < \nu_1 + \nu_2 < 1$), and gives a negative amplitude, hence a repulsive interaction. For the magnetic or magnetically dominating (in the sense mentioned earlier) cases, the nature of interaction will keep hold even at a general separation. However, the nature of force will become indefinite at small separation when the effects of fluxes are electric or electrically dominant. The basic feature about the nature of interaction on the brane-separation remains similar to our earlier discussion for systems with $p = p'$ considered in [4, 2].

We now move to discuss the analytical structure of amplitudes at small separation Y for which the open string description is appropriate. This can be achieved via the Jacobian transformation of the integration variable $t \to t' = 1/t$, converting the tree-level closed string cylinder diagram to the open string one-loop annulus diagram. In terms of this annulus variable t', noting

$$\eta(\tau) = \frac{1}{(-i\tau)^{1/2}} \eta\left(-\frac{1}{\tau}\right), \qquad \theta_1(\nu|\tau) = i \frac{e^{-i\pi\nu^2/\tau}}{(-i\tau)^{1/2}} \theta_1\left(\frac{\nu}{\tau}\Big| -\frac{1}{\tau}\right), \tag{14}$$

the amplitude in each class can be explicitly re-expressed, respectively, as

$$\begin{aligned}\Gamma_{\mathrm{I}} \;=\; & -\frac{2\, n_p n_{p-2}\, V_{p-1} C_F \sin\pi\nu}{(8\pi^2\alpha')^{\frac{p-1}{2}}} \int_0^\infty dt' e^{-\frac{Y^2 t'}{2\pi\alpha'}} t'^{\frac{1-p}{2}} \frac{\left[\cos(-i\pi\nu t') - \cosh\frac{\pi t'}{2}\right]^2}{\sin(-i\pi\nu t')\sin(-i\pi t'/2)} \\ & \times \prod_{n=1}^{\infty} \frac{1}{(1-|z|^{2n})^4} \prod_{j=1}^{2} \frac{\left(1 - e^{\pi(\nu + (-)^j/2)t'}|z|^{2n}\right)^2 \left(1 - e^{-\pi(\nu+(-)^j/2)t'}|z|^{2n}\right)^2}{\left(1 - e^{(-)^j 2\pi\nu t'}|z|^{2n}\right)\left(1 - e^{(-)^j \pi t'}|z|^{2n}\right)}, \end{aligned} \tag{15}$$

$$\begin{aligned}\Gamma_{\rm II} &= -\frac{2\,n_p n_{p-2}\,V_{p-1}\tan\pi\nu_1\tan\pi\nu_2}{(8\pi^2\alpha')^{\frac{p-1}{2}}}\int_0^\infty dt' e^{-\frac{Y^2 t'}{2\pi\alpha'}} t'^{\frac{3-p}{2}}\\
&\times\frac{\left[\cos\left(-i\pi(\nu_1-\nu_2)t'\right)-\cosh\frac{\pi t'}{2}\right]\left[\cos\left(-i\pi(\nu_1+\nu_2)t'\right)-\cosh\frac{\pi t'}{2}\right]}{\sin(-i\pi\nu_1 t')\sin(-i\pi\nu_2 t')\sinh(\pi t'/2)}\prod_{n=1}^\infty\frac{1}{(1-|z|^{2n})^2}\\
&\times\prod_{k=1}^2\frac{1}{1-e^{(-)^k\pi t'}|z|^{2n}}\prod_{j=1}^2\frac{1-2|z|^{2n}\cosh[\pi(\nu_1+(-)^j\nu_2+\frac{(-)^k}{2})t']+|z|^{4n}}{1-e^{(-)^k 2\pi\nu_j t'}|z|^{2n}},\end{aligned}\tag{16}$$

$$\begin{aligned}\Gamma_{\rm III} &= -\frac{2n_p n_{p-2}V_{p-1}\tan\pi\nu_1}{(8\pi^2\alpha')^{\frac{p-1}{2}}}\int_0^\infty dt' e^{-\frac{Y^2 t'}{2\pi\alpha'}} t'^{\frac{1-p}{2}}\frac{[\cos(-i\pi\nu_1 t')-\cos(-i\pi\nu_2 t')]^2}{\sin(-i\pi\nu_1 t')\sin(-i\pi\nu_2 t')}\\
&\times\prod_{n=1}^\infty\frac{1}{(1-|z|^{2n})^4}\prod_{j=1}^2\frac{\left(1-e^{\pi(\nu_1+(-)^j\nu_2)t'}|z|^{2n}\right)^2\left(1-e^{-\pi(\nu_1+(-)^j\nu_2)t'}|z|^{2n}\right)^2}{(1-e^{2\pi\nu_j t'}|z|^{2n})\,(1-e^{-2\pi\nu_j t'}|z|^{2n})},\end{aligned}\tag{17}$$

with now $|z| = e^{-\pi t'}$. When $\nu = \nu_0$ in Class I with $0 < \nu_0 < 1/2$ or $\nu_j = \nu_{j0}$ with $0 < \nu_{j0} < 1/2$ ($j = 1, 2$) in Class II or with $0 < \nu_{10} < 1/2$ and $0 < \nu_{20} < 1$ in Class III, the underlying amplitude remains real and will diverge when the separation $Y \le \pi\sqrt{2(1/2-\nu)\alpha'}$ in Class I, or $Y \le \pi\sqrt{2(1/2-\nu_1-\nu_2)\alpha'}$ when $0 < \nu_1+\nu_2 < 1/2$ in Class II, or $Y \le \pi\sqrt{2|\nu_1-\nu_2|\alpha'}$ in Class III when $\nu_1 \ne \nu_2$, i.e., with each on the order of string scale. Along with a similar discussion given in [4, 2, 8, 9], this divergence indicates the onset of tachyonic instability in each respective case, giving rise to the relaxation of the underlying system to form the final stable bound state. When $1/2 < \nu_1+\nu_2 < 1$ in Class II, the force as mentioned earlier is repulsive and becomes larger when the separation Y becomes smaller and for this reason the integrand has no exponential blow-up singularity to show up even at $t' \to \infty$, i.e., no onset of tachyonic singularity, for any $Y > 0$. Note that if we express the amplitude (17) in Class III above in terms of $p' = p-2$, i.e., the spatial NN-directions, its structure looks similar to the one given in [2] for $p = p'$ with the respective two fluxes not sharing any common index. So many of the underlying properties such as the onset of various instabilities remain the same as those given in [2], therefore referred there for detail. For this reason, we discuss the remaining new features only below for this Class.

We move to discuss the more rich structure and the associated physics when $\nu = i\nu_0$ with $0 < \nu_0 < \infty$ in Class I or $\nu_1 = i\nu_{10}$ with $0 < \nu_{10} < \infty$ and $\nu_2 = \nu_{20}$ with ν_{20} real in Class II[8] or III . In Class I, this corresponds to the presence of at least one electric flux

[8]The case with ν_1 real and ν_2 imaginary can be similarly discussed in this class.

(or being electrically dominant) along a NN-direction. Now the integrand has an infinite number of simple poles occurring on the positive real axis at $t' = k/\nu_0$ with $k = 1, 2, \cdots$ and for this, as discussed in [10, 11, 12, 13, 14, 15, 4, 2], the amplitude has an imaginary part which is sum of the residues at these simple poles and gives rise to the rate of pair production of open strings. This rate per unit $(p-1)$-worldvolume is

$$\begin{aligned}
\mathcal{W}_{\rm I} \equiv -\frac{2{\rm Im}\Gamma_{\rm I}}{V_{p-1}} &= \frac{4 n_p n_{p-2} C_F \sinh \pi\nu_0}{\nu_0 (8\pi^2\alpha')^{\frac{p-1}{2}}} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{\nu_0}{k}\right)^{\frac{p-1}{2}} e^{-\frac{kY^2}{2\pi\nu_0\alpha'}} \frac{\left[\cosh \frac{k\pi}{2\nu_0} - (-)^k\right]^2}{\sinh \frac{k\pi}{2\nu_0}} \\
&\times \prod_{n=1}^{\infty} \frac{\left[1 - 2(-)^k e^{-\frac{2nk\pi}{\nu_0}} \cosh \frac{\pi k}{2\nu_0} + e^{-\frac{4nk\pi}{\nu_0}}\right]^4}{\left(1 - e^{-\frac{2nk\pi}{\nu_0}}\right)^6 \left(1 - 2 e^{-\frac{2nk\pi}{\nu_0}} \cosh \frac{\pi k}{\nu_0} + e^{-\frac{4nk\pi}{\nu_0}}\right)}.
\end{aligned} \tag{18}$$

This rate shares the common features as found in similar cases studied in [2] when $p = p'$ with the two fluxes sharing at least one common index. Namely, it is suppressed by the separation Y but enhanced by the value of ν_0, which can be determined by flux/fluxes via $\cosh \pi\nu_0 = \cos \pi\nu$ as given Table 1. Each term in the sum is suppressed by the integer k but diverges as the electric field reaches its critical value, i.e., $\nu_0 \to \infty$, signaling the onset of a singularity [14]. However, the present rate differs in many aspects: the number of terms in the sum doubles and the contribution to the rate is positive for odd k and negative for even k. In particular, there is an enhanced factor $[\cosh \frac{k\pi}{2\nu_0} - (-)^k]^2/\sinh \frac{k\pi}{2\nu_0}$ which becomes important for small ν_0. This can be examined easily by looking at the leading order approximation, i.e., the $k = 1$ term in the sum as

$$(2\pi\alpha')^{(p'+1)/2} \mathcal{W}_{\rm I} \approx 2\pi n_p n_{p-2} C_F \left(\frac{\nu_0}{4\pi}\right)^{(p-1)/2} e^{-\frac{Y^2 - \pi^2\alpha'}{2\pi\nu_0\alpha'}}, \tag{19}$$

where C_F is given in Table 1 and the small ν_0 can be explicitly determined, to leading order, in terms of the fluxes present. Note that small ν_0 does need all fluxes along NN directions, electric or magnetic, to be small. Let us consider the first case in Table 1 as an illustration for which $C_F \approx 1$. Now $\cosh \nu_0 = \cos \nu = (1 - f_1 f_2)/\sqrt{(1 - f_1^2)(1 - f_2^2)}$ which gives, to leading order, $\nu_0 \approx |f_1 - f_2|/\pi$. Since $p \geq 3$, the rate is largest for $p = 3$ for given C_F and small ν_0 (for fixed n_p and n_{p-2}) and can be significant at a separation $Y = \pi\sqrt{\alpha'} + 0^+$, i.e. on the order of string scale, where we don't have yet the onset of tachyonic instability which occurs for $Y \leq \pi\sqrt{\alpha'}$ from the real part of the amplitude. This is in spirit similar to the enhanced rate discussed in [2], for which $p = p'$ with $p = p' = 3$ giving the largest rate, but for a completely different case where a reasonably large magnetic flux not sharing any common index with the weak electric flux must be present. The novel feature here is that the $p'\,(= p - 2)$ branes play effectively as such a magnetic

flux and this is rational for the enhancement even in the absence of a magnetic flux. As discussed above, for small ν_0, only the magnetic flux (electric flux) with one index along a DN direction can further enhance (reduce) the rate through $C_F \approx \sqrt{1+f^2}\,(\approx \sqrt{1-f^2})$ with f the magnetic (electric) flux. Note that the magnetic flux $|f|$ is measured in string units and for a realistic value, $|f|$ should be smaller than unity, therefore giving $C_F \approx 1$. In other words, the enhancement due to a magnetic flux in general is small and so we can ignore this for Class I from now on. Let us make some numerical estimation of this rate for small ν_0 and this may serve for sensing its significance. For this purpose, we take $n_p = n_{p-2} = 5$, $\nu_0 = 0.02$ and $C_F \approx 1$. We also take the brane separation as given above such that the exponential in the rate can be approximated to one. So the rate in string units is $(2\pi\alpha')^{(p'+1)/2}\mathcal{W}_I = 2\pi n_{p-2} n_p (\nu_0/4\pi)^{(p-1)/2} = 0.25, 0.02$ for $p = 3, 4$, respectively. The largest rate occurs indeed at $p = 3$, the rate for $p = 4$ is one order of magnitude smaller and the larger the p the smaller the rate. This estimation indicates that the rate for $p = 3$ can indeed be significant for small separation, for reasonably chosen n_p and n_{p-2}, and even for small fluxes, therefore more realistic than the case mentioned above in [2]. The small-separation also implies that the significantly produced open string pairs are almost confined on the branes along the electric flux line. This further implies that the radiations due to the annihilation of the open string pairs in a short time should be mostly along the brane directions. If string theories are relevant, given the large rate for $p = 3$, we expect that the early Universe or even macroscopic objects in the sky at present can give rise to such open string pair production, therefore large radiations, which may have potential observational consequence. This may further imply why our world has three large spatial dimensions since the observational consequence if any can only be significant for $p = 3$. Pursuing any of these possibilities is beyond the scope of present work and we expect to examine carefully some of these possibilities in the future.

We have also learned from the above that we need at least one electric flux being along the NN-direction to give rise to the pair production. This is clearly indicated in the case of $(\alpha, \beta) = (0, a), (\gamma, \delta) = (c, d), a \in \mathrm{DN}, c, d \in \mathrm{NN}$ given in Table 1 for which the electric flux is along a spatial DN-direction and as such $\nu = \nu_0$ is real (therefore no pair production of open strings), depending only on the magnetic flux. The triviality of the electric flux in this case can be understood via a T-duality along the electric flux direction. The D_p branes then become D_{p-1} branes while the $\mathrm{D}_{p'}$ become $\mathrm{D}_{p'+1}$ with the D_{p-1} moving with a velocity, determined by the original electric flux along the T-dual direction, relative to the $\mathrm{D}_{p'+1}$ in the T-dual picture. Since the $\mathrm{D}_{p'+1}$ has a Lorentz symmetry along the T-dual direction, so such a relative motion can be removed by a Lorentz boost along this direction. If we now T-dual back, we end up with a system without the presence of an

electric flux but with an additional overall factor in the amplitude due to the boost.

For Class II or III, there is only one case relevant (see footnote 8 for Class II) for which $\nu_1 = i\nu_{10}$ is imaginary ($0 < \nu_{10} < \infty$) while $\nu_2 = \nu_{20}$ is real with $0 < \nu_{20} < 1/2$ in Class II and with $0 < \nu_{20} < 1$ in Class III. This corresponds to $(\alpha, \beta) = (a, b), (\gamma, \delta) = (0, c)$ with $c \in$ NN, i.e., the electric flux along a NN-direction and in Class II, $c \neq a, b \in$ NN (see Table 2) while $a, b \in$ DN in Class III (see Table 3). In either case, the integrand has also an infinite number of simple poles occurring on the positive real axis at $t' = k/\nu_{10}$ with $k = 1, 2, \cdots$. By the same token as in Class I above, the pair production rate for the case in Class II ($p \geq 5$) is

$$\mathcal{W}_{\rm II} = \frac{4 n_p n_{p-2} \tanh \pi\nu_{10} \tan \pi\nu_{20}}{\nu_{10}(8\pi^2\alpha')^{\frac{p-1}{2}}} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{\nu_{10}}{k}\right)^{\frac{p-3}{2}} \frac{\left[(-)^k \cosh \frac{\pi\nu_{20} k}{\nu_{10}} - \cosh \frac{\pi k}{2\nu_{10}}\right]^2}{\sinh \frac{\pi\nu_{20} k}{\nu_{10}} \sinh \frac{\pi k}{2\nu_{10}}}$$
$$\times e^{-\frac{Y^2 k}{2\pi\nu_{10}\alpha'}} \prod_{n=1}^{\infty} \frac{1}{(1 - e^{-\frac{2nk\pi}{\nu_{10}}})^4} \prod_{j=1}^{2} \frac{\left[1 - 2(-)^k e^{-\frac{2nk\pi}{\nu_{10}}} \cosh \frac{\pi k}{\nu_{10}} \left(\nu_{20} + \frac{(-)^j}{2}\right) + e^{-\frac{4nk\pi}{\nu_{10}}}\right]^2}{\left(1 - e^{-\frac{k\pi}{\nu_{10}}(2n + (-)^j)}\right)\left(1 - e^{-\frac{2nk\pi}{\nu_{10}}(n + (-)^j \nu_{20})}\right)} \tag{20}$$

This case looks in almost every aspect, for examples, the onset of various instabilities for both real and imaginary parts of the amplitude, even closer to the one in [2] with the two fluxes not sharing any common index, and for this reason not repeating here, except for one subtle point regarding the enhancement factor for the pair production rate given above. Let us specify this for small ν_{10} (for fixed non-vanishing ν_{20}) for which the rate can be approximated by the leading $k = 1$ term as

$$(2\pi\alpha')^{(p'+1)/2} \mathcal{W}_{\rm II} \approx n_p n_{p-2} \left(\frac{\nu_{10}}{4\pi}\right)^{(p'-1)/2} e^{-\frac{Y^2}{2\pi\nu_{10}\alpha'}} e^{\frac{\pi(1-2\nu_{20})}{2\nu_{10}}} \tan \pi\nu_{20}. \tag{21}$$

If we focus on the NN-directions, i.e., $p' = p-2 \geq 3$, the dimensionless rate $(2\pi\alpha')^{(p'+1)/2}\mathcal{W}_{\rm II}$ differs from the first equality given in Eq.(87) in [2] only in the enhancement factor and the ratio of the two is $e^{\frac{\pi(1-4\nu_{20})}{2\nu_{10}}} > 1$ for $0 < \nu_{20} < 1/4$ and is less than unity for $1/4 < \nu_{20} < 1/2$. For $\nu_{20} \ll \nu_{10}$, i.e., vanishing magnetic flux, the rate looks identical to the one given in Eq.(19) above in Class I but with now $p \geq 5$ and C_F set to unity, with again the $\mathrm{D}_{p'}$ as an effective magnetic flux mentioned earlier.

For the case in Class III,

$$\mathcal{W}_{\rm III} = \frac{4 n_p n_{p-2} \tanh \pi\nu_{10}}{\nu_{10}} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{\nu_{10}}{8k\pi^2\alpha'}\right)^{\frac{p-1}{2}} e^{-\frac{kY^2}{2\pi\nu_{10}\alpha'}} \frac{\left[\cosh \frac{k\pi\nu_{20}}{\nu_{10}} - (-)^k\right]^2}{\sinh \frac{k\pi\nu_{20}}{\nu_{10}}}$$
$$\times \prod_{n=1}^{\infty} \frac{\left[1 - 2(-)^k e^{-\frac{2nk\pi}{\nu_{10}}} \cosh \frac{k\pi\nu_{20}}{\nu_{10}} + e^{-\frac{4nk\pi}{\nu_{10}}}\right]^4}{\left[1 - e^{-\frac{2nk\pi}{\nu_{10}}}\right]^6 \left[1 - e^{-\frac{2k\pi}{\nu_{10}}(n-\nu_{20})}\right] \left[1 - e^{-\frac{2k\pi}{\nu_{10}}(n+\nu_{20})}\right]}, \tag{22}$$

which again shares many common features mentioned above in Class II as the one given in [2]. For small ν_{10}, the rate can be approximated by the leading $k=1$ term as

$$(2\pi\alpha')^{(p'+1)/2}\mathcal{W}_{\rm III} = 2\pi n_p n_{p-2}\left(\frac{\nu_{10}}{4\pi}\right)^{(p-1)/2} e^{-Y^2/2\pi\nu_{10}\alpha'} e^{\pi\nu_{20}/\nu_{10}}, \tag{23}$$

with $p = p' + 2 \geq 3$. This dimensionless rate is largest for $p = 3$ (or $p' = 1$) with an enhancement factor $2\pi e^{\pi\nu_{20}/\nu_{10}}$ vs $e^{\pi\nu_{20}/\nu_{10}}\tan\pi\nu_{20}$ given in [2]. Apart from a difference of a factor of $\tan\pi\nu_{20}$, the nature and range of ν_{20} here are completely different[9]. In the present case, $0 < \nu_{20} < 1$ with $\nu_{20} = 1/2$ corresponding to vanishing magnetic flux, $\nu_{20} \to 0$ when the magnetic flux $f_2 \to -\infty$ and $\nu_{20} \to 1$ when $f_2 \to \infty$. So once again even without the presence of a magnetic flux, i.e., $\nu_{20} = 1/2$, we still have an exponential enhancement of the rate and the rate in each Class looks identical in the absence of a magnetic flux. The various implications of the rate in class II or III can be similarly discussed as in Class I above and will not be repeated, except for one point to which we turn next. The largest rate in Class II occurs for $p' = 3$ and $p = 5$. So if there is indeed an observational consequence, this may indicate the existence of large extra dimensions since $p = 5$ other than the usual $p = 3$ in this case.

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References

[1] P. Di Vecchia and A. Liccardo, "D branes in string theory. I," NATO Adv. Study Inst. Ser. C. Math. Phys. Sci. **556**, 1 (2000) [arXiv:hep-th/9912161].

[2] J. X. Lu, and S. S. Xu, "The open string pair-production rate enhancement," [arXiv:0904.4112 [hep-th]].

[9]In [2], $\nu_{20} \to 0$ when the magnetic flux $f_2 \to 0$ and $\nu_{20} \to 1/2$ when $|f_2| \to \infty$.

[3] J. Polchinski, "Superstring Theory", Vol. 2, Cambridge: Cambridge University Press (1998)

[4] J. X. Lu, B. Ning, R. Wei and S. S. Xu, "Interaction between two non-threshold bound states," Phys. Rev. D **79**, 126002 (2009) [arXiv:0902.1716 [hep-th]].

[5] P. Di Vecchia, M. Frau, A. Lerda and A. Liccardo, "(F,Dp) bound states from the boundary state," Nucl. Phys. B **565**, 397 (2000) [arXiv:hep-th/9906214].

[6] M. Billo, P. Di Vecchia, M. Frau, A. Lerda, I. Pesando, R. Russo and S. Sciuto, "Microscopic string analysis of the D0-D8 brane system and dual R-R states," Nucl. Phys. B **526**, 199 (1998) [arXiv:hep-th/9802088].

[7] E. T. Whittaker and G. N. Watson, "A Course of Modern Analysis", 4th Ed (reprinted). Cambridge: Cambridge University Press (1963)

[8] T. Banks and L. Susskind, "Brane - Antibrane Forces," arXiv:hep-th/9511194.

[9] J. X. Lu, B. Ning, S. Roy and S. S. Xu, "On Brane-Antibrane Forces," JHEP **0708**, 042 (2007) [arXiv:0705.3709 [hep-th]].

[10] C. Bachas and M. Porrati, "Pair creation of open strings in an electric field," Phys. Lett. B **296**, 77 (1992) [arXiv:hep-th/9209032].

[11] C. Bachas, "Schwinger effect in string theory," arXiv:hep-th/9303063.

[12] M. B. Green and M. Gutperle, "Light-cone supersymmetry and D-branes," Nucl. Phys. B **476**, 484 (1996) [arXiv:hep-th/9604091].

[13] J. H. Cho, P. Oh, C. Park and J. Shin, "String pair creations in D-brane systems," JHEP **0505**, 004 (2005) [arXiv:hep-th/0501190].

[14] M. Porrati, "Open strings in constant electric and magnetic fields," arXiv:hep-th/9309114.

[15] B. Chen, and X. Liu, "D1-D3 (or $\overline{D}_3$) Systems with Fluxes," JHEP **0808**, 034 (2008) [arXiv:0806.3548 [hep-th]].